PG – 606

I Semester M.Sc. Degree Examination, January 2015
(CBCS Scheme)
MATHEMATICS
M101T : Algebra – I

Time : 3 Hours Max. Marks : 70

***Instructions**: 1) Answer **any 5** questions.*
*2) **All** questions carry **equal** marks.*

1. a) Let $\phi : G \to \overline{G}$ be an epimorphism with Kernel K and let N be a normal subgroup of G. Then prove that $\dfrac{G/K}{N/K} \approx G/N$. **5**

b) Let G be an abelian group with $a \in G$ such that $a^2 \neq e$. Prove that there is an automorphism of G different from I. **4**

c) State and prove the Cayley's theorem for permutation groups. **5**

2. a) State and prove the orbit stabilizer theorem. **5**

b) If G is a finite group, then show that $o(G) = o(Z(G)) + \sum_{N(a) \neq G} \dfrac{o(G)}{o(N(a))}$. **5**

c) Prove that every group of prime order has non trivial centre. **4**

3. a) If A and B are two subgroups of a finite group G, then prove that $o(A \times B) = \dfrac{o(A)\, o(B)}{o(A \cap xBx^{-1})}$, where $A \times B = \{a\, x\, b / a \in A, b \in B\}$. Hence show that all p-sylow sub groups of a finite group are conjugate to each other. **8**

b) Show that a group of order pq with p and q are distinct primes such that p < q and $q \not\equiv$ (mod p) is abelian. **6**

4. a) Show that the symmetric group S_3 is not simple. **3**

b) Define a solvable group. Give an example of a non-abelian solvable group. **3**

c) State and prove the Jordan-Hölder theorem. **8**

P.T.O.

5. a) Let R be a commutative ring with unity whose only ideals are {o} and R itself. Then prove that R is a field. **5**

b) Let U be a left ideal of a ring R and $\lambda(U) = \{x \in R \mid xu = 0 \; \forall \; u \in U\}$. Then prove that $\lambda(U)$ is an ideal of R. **4**

c) Let R and R' be rings and ϕ is a homomorphism of R onto R' with Kernel U. Then show that $R' \approx R/U$. **5**

6. a) Define a principle ideal ring. Show that a ring Z of integers is a principle ideal ring. **4**

b) Prove that an ideal of the ring Z of integers is maximal if and only if it is generated by some prime integer in Z. **5**

c) Show that any two isomorphic integral domains have isomorphic quotient fields. **5**

7. a) Define Euclidean ring. Prove that the ring Z [i] of Gaussian integers is an Euclidean ring. **4**

b) Show that every Euclidean ring is a principle ideal ring. **4**

c) State and prove the Unique factorization theorem. **6**

8. a) Show that the product of two primitive polynomials is a primitive polynomial. **5**

b) State and prove the Gauss lemma. **5**

c) Using Eisenstein Criteria, verify that the polynomial $x^3 - 3x - 1$ over Q is irreducible. **4**